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TUTORIAL WORKSHOP TITLE

Fractional Order Calculus and Its Applications in Mechatronic System Controls

TUTORIAL WORKSHOP WEB:

http://mechatronics.ece.usu.edu/foc/ieee-icma06-tutorial/

WORKSHOP ORGANIZERS (see below for bios)

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YangQuan Chen – Utah State University, USA

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WORKSHOP DURATION: Half day

LIST OF PRESENTERS

Dingyu Xue - Northeastern Universtiy

YangQuan Chen – Utah State University, USA

WORKSHOP ABSTRACT AND OBJECTIVES

The purpose of this tutorial workshop is to introduce the fractional calculus and its applications in controller designs. Fractional order calculus, or integration and differentiation of an arbitrary order or fractional order, is a new tools that extends the descriptive power of the conventional calculus. The tools of fractional calculus support mathematical models that in many cases more accurately describe the dynamic response of actual systems in electrical, mechanical, and automatic control applications etc. The theoretical and practical interest of these fractional order operators is nowadays well established, and its applicability to science and engineering can be considered as emerging new topics. The need to digitally compute the fractional order derivative and integral arises frequently in many fields especially in automatic control and digital signal processing. Fractional order proportional-integral-derivative (PID) controllers are based on the fractional order calculus where the derivative or integral can be of a non-integer order. Due to the extra tuning knobs, it is expected that better control performance can be achieved if the fractional order PID controller is used. Fractional calculus has much to offer science and engineering by providing not only new mathematical tools, but more importantly, its application suggests new insights into the system dynamics as well as controls.

WORKSHOP AUDIENCE

The expected audience includes engineers, scientists, postgraduate students, and academics. The workshop will be self-contained so that it is suitable for systems and control researchers and practitioners who may not be familiar with the concept of fractional order systems as well as to those with some initial background in the field.

ORGANIZATIONAL DETAILS

1. Attendees will be given hardcopies of the presentation slides and will be provided electronic copies as well.
2. The organizers and presenters will use electronic projection in PowerPoint or PDF format. We will provide our own computers, but we require the use of an LCD projector.
3. We would prefer to not have more than 50 attendees.

ORGANIZERS' and PRESENTERS' BIOGRAPHIES

Dingyu Xue (Organizer) received the B.S. and M.S. degrees in automatic control engineering from Shenyang Polytechnic University and Northeastern University, respectively. He received the D.Phil. in control engineering from Sussex University. He took up teaching posts in Northeastern University since 1993, and was appointed the professor in the Faculty of Information Sciences and Engineering in Northeastern University since 1997. He is the author and coauthor of many monographs and textbooks from Tsinghua University Press, mainly concentrate on computer aided control systems design, system simulation and MATLAB based mathematical problem solutions.

YangQuan Chen (co-Organizer) is presently an assistant professor of Electrical and Computer Engineering Department and the Acting Director for CSOIS (Center for Self-Organizing and Intelligent Systems) at Utah State University. He obtained his Ph.D. from Nanyang Tech. Univ. (NTU), Singapore. Dr Chen has 12 US patents granted and 2 US patent applications published, most related to iterative learning control and repetitive control. He published over 160 academic papers, two textbooks, one research monograph, and (co)authored over 50 industrial reports. He has been an Associate Editor in the Conference Editorial Board of IEEE Control Systems Society since 2002 and is a founding member of the ASME subcommittee of "Fractional Dynamics" in 2003. He is a senior member of IEEE, a member of ASME and a member of International Society for Information Fusion.

WORKSHOP PROGRAM

The workshop begins with an introduction to the theoretical foundation of FOS including fractional calculus and its basic properties. It is now well known that many physical phenomena can be modeled accurately and effectively using fractional derivatives, whereas the integral derivative based models capture these phenomena only approximately. It has been demonstrated that many fractional derivatives based controls are far superior than integer order derivative based control schemes. After this introduction, approaches to digital computation of fractional order derivative and integral will be presented. A new synthesis approximation method is introduced to approximate fractional order differentiator operator s^{α} in a given frequency range of interest and realize frequency-band limited FOS. This new method can achieve better approximation than the existent discretization methods in the whole frequency range. Next, it is shown how the building Simulink model can be used to obtain numerical solution of fractional order calculus equation. We will illustrate that this Simulink model makes it possible to resolve the fractional order linear and nonlinear calculus equation.

Next we will begin to consider design and apply the FOC. According to the desired gain margin and phase margin, the designed fractional order PID controller can meet the stability robustness of the feedback control loop. Designed FOC can be applied to the integer order system and fractional order system. Then we apply the FOC to a basic control problem in automatic control — position servomechanism control system. The better robustness performances of FOC are shown by the comparison of fractional order PID control’s responses with normal PID control.

WORKSHOP SCHEDULE

Time	Topic	Presenter
14:00-14:10	Welcome and Introduction	D Xue
14:10-14:55	Brief tutorial and historical review of fractional calculus and overview of its applications	Y. Q. Chen
14:55-15:40	Computer solutions to fractional calculus problems	D Xue
15:40-16:00	Break	
16:00-16:45	Applications of fractional order controllers design in control systems	D Xue
16:45-17:30	Fractional order controller versus integer order controllers: comparative studies through benchmark problems	D. Xue
17:30-17:40	Closing remarks (what’s hot and what’s next)	D. Xue and Y. Q. Chen

DETAILED WORKSHOP SYNOPSIS

Brief historic review of fractional calculus and overview of its applications

There is an increasing interest in dynamic systems of non-integer orders. Extending derivatives and integrals from integer orders to non-integer orders has a firm and long standing theoretical foundation. For example, Leibniz mentioned this concept in a letter to L'Hospital over three hundred years ago and the earliest more or less systematic studies have been made in the beginning and middle of the 19th century by Liouville, Riemann and Holmgren. In the literature, people often use the term "*fractional* order calculus", or "*fractional* order dynamic system" where "*fractional*" actually means "*non-integer*".

Among the literature, the definitions commonly used are the Riemann-Liouville definition and the Grünwald-Letnikov definition, which are given below

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^m \left(\int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}}\mathrm{d}\tau\right) $$

$$ {}_aD_x^{\alpha}[f(t)] = \lim_{h\to 0}\frac{1}{\Gamma(\alpha)h^{\alpha}}\sum_{k=0}^{[(x-a)/h]}\frac{\Gamma(\alpha+k)}{\Gamma(k+1)} f(t-kh) $$

With such definitions, the fractional order calculus are established and used in many fields. Control theory is one of the areas, where fractional order calculus used widely. Many originally considered lumped systems can be more exactly described by fractional order systems. And in controller design, fractional order controllers also exhibit their advantages. For instance, in fractional order PID controllers, $G_c(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu}$, two extra tuning knobs, λ and μ, make the controller more flexible and the behaviors of the controller may superior to conventional controllers.

Computer solutions to fractional calculus problems

The computation of the fractional order calculus is very useful in the studies of controller design. In the tutorial workshop, systematic methods and MATLAB based computer routines will be given, where

- Off-line fractional order differentiators using Grünwald-Letnikov definition
- On-line approximation of differentiators with improved Oustaloup's filters where

$$ s^{\alpha} \approx K\left(\frac{ds^2 + bs\omega_h}{d(1-\alpha)s^2 + bs\omega_h + d\alpha}\right)\prod_{k=-N}^{N}\frac{1+s/\omega_k'}{1+s/\omega_k} $$

where

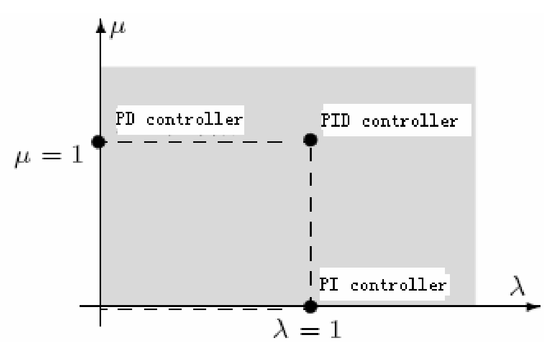
$$ \omega_k' = \left(\frac{d}{b\omega_h}\right)^{\frac{\alpha-2k}{2N+1}}, \quad \omega_k = \left(\frac{b}{d\omega_b}\right)^{\frac{\alpha+2k}{2N+1}}, \quad K = \left(\frac{d\omega_b}{b}\right)^{\alpha}\prod_{k=-N}^{N}\frac{\omega_k}{\omega_k'} $$

- Exact solutions to fractional order linear differential equations
- Block-diagram based algorithms for fractional order nonlinear differential equations
- Frequency domain analysis to fractional order linear systems.
- Realization and discretization for fractional order systems
- Properties of fractional order systems

Applications of fractional order controllers design in control systems

Fractional order PID controller is a very good example for process control systems, whose mathematical model is given by

$$ G_c(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu} $$



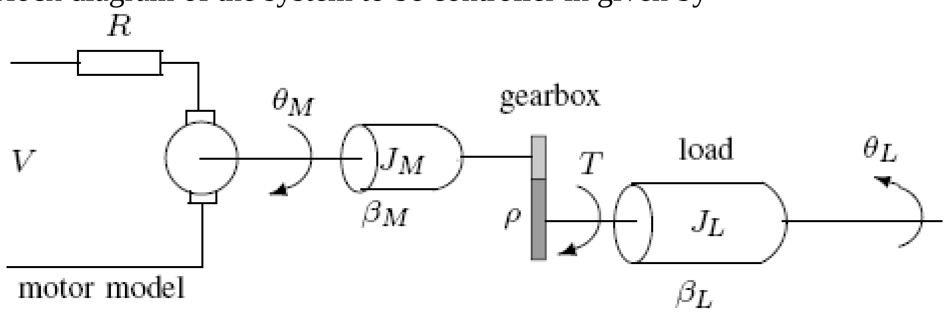
where two extra knobs are provided. It can be seen from the figure that, the conventional PI controller, PD controller and PID controllers are special cases of fractional order PID controllers. Flexibility isare provided in the design of such controllers. Better performance can be expected in using such controllers. It will be shown in the next section that the fractional PID controller may have superior properties to the integer order PID controllers. The controller design techniques given in this part are:

- Introduction to fractional order PID controllers
- Discussions in performance index selections in controller design.
- ITAE criterion based optimum controller design technique.
- An optimum order controller designer and its applications in controller design.

Fractional order controller versus integer order controllers: comparative studies through benchmark problems

A well-established position servomechanism is used as a benchmark problem in the comparisons of best integer order PI/PID controllers and the best fractional order PI/PID controllers. Comparisons will be made through different aspect. The control performance will be compared first, and the robustness of the controllers to mechanical nonlinearities, the robustness to elastic parameter changes, and so on, will be given.

The block diagram of the system to be controller in given by



The mathematical model of the system is given as

$$\dot{x}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_\theta}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{\rho J_M} & 0 & -\frac{k_\theta}{\rho^2 J_M} & -\frac{\beta_M + k_T^2/R}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{RJ_M} \end{bmatrix} V$$

For this benchmark model, the robustness is very high as demonstrated below